

Atmospheric neutrinos

Analytic/numerical methods

Outline

- Introduction
 - Calculating the flux of atmospheric neutrinos
 - Primary cosmic-ray spectrum
- Muon charge ratio, K/π ratio, $\nu/\bar{\nu}$ ratio
- Atmospheric neutrinos to PeV
 - Must account for knee in cosmic-ray spectrum
- Atmospheric muon veto

Solution of cascade equations for μ & ν

Same form for μ ;
Different kinematics
 $\rightarrow \mu, \nu$ differences

$$\phi_\nu(E_\nu) = \phi_N(E_\nu) \times \left\{ \frac{A_{\pi\nu}}{1 + B_{\pi\nu} \cos(\theta) E_\nu / \epsilon_\pi} + \frac{A_{K\nu}}{1 + B_{K\nu} \cos(\theta) E_\nu / \epsilon_K} + \frac{A_{\text{charm}\nu}}{1 + B_{\text{charm}\nu} \cos(\theta) E_\nu / \epsilon_{\text{charm}}} \right\},$$

$$\begin{aligned}\epsilon_\pi &= 115 \text{ GeV} \\ \epsilon_K &= 850 \text{ GeV} \\ \epsilon_{\text{charm}} &> 10 \text{ PeV}\end{aligned}$$

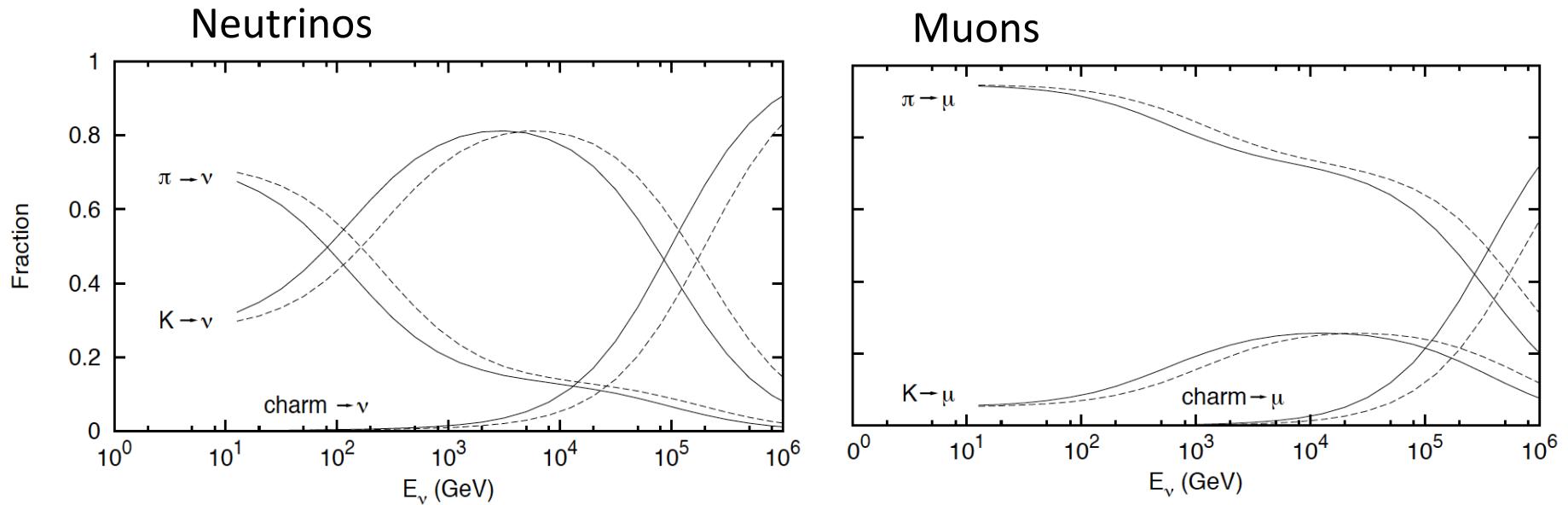
$$A_{i\nu} = \frac{Z_{Ni} \times BR_{i\nu} \times Z_{i\nu}}{1 - Z_{NN}} \quad Z_{pK^+} = \frac{1}{\sigma} \int x^\gamma \frac{d\sigma(x)}{dx} dx$$

$$Z_{\pi\mu} = \frac{1 - r_\pi^{\gamma+1}}{(\gamma + 1)(1 - r_\pi)} \quad \text{and} \quad \frac{\epsilon_\pi}{\cos \theta E_\mu} \frac{1 - r_\pi^{\gamma+2}}{(\gamma + 2)(1 - r_\pi)}$$

$$Z_{\pi\nu} = \frac{(1 - r_\pi)^\gamma}{(\gamma + 1)} \quad \text{and} \quad \frac{\epsilon_\pi}{\cos \theta E_\mu} \frac{(1 - r_\pi)^{(\gamma+1)}}{(\gamma + 2)} \quad \begin{aligned}r_\pi &= 0.573 \quad \text{but} \\ r_K &= 0.0458\end{aligned}$$

Kinematic differences for μ, ν (K, π)

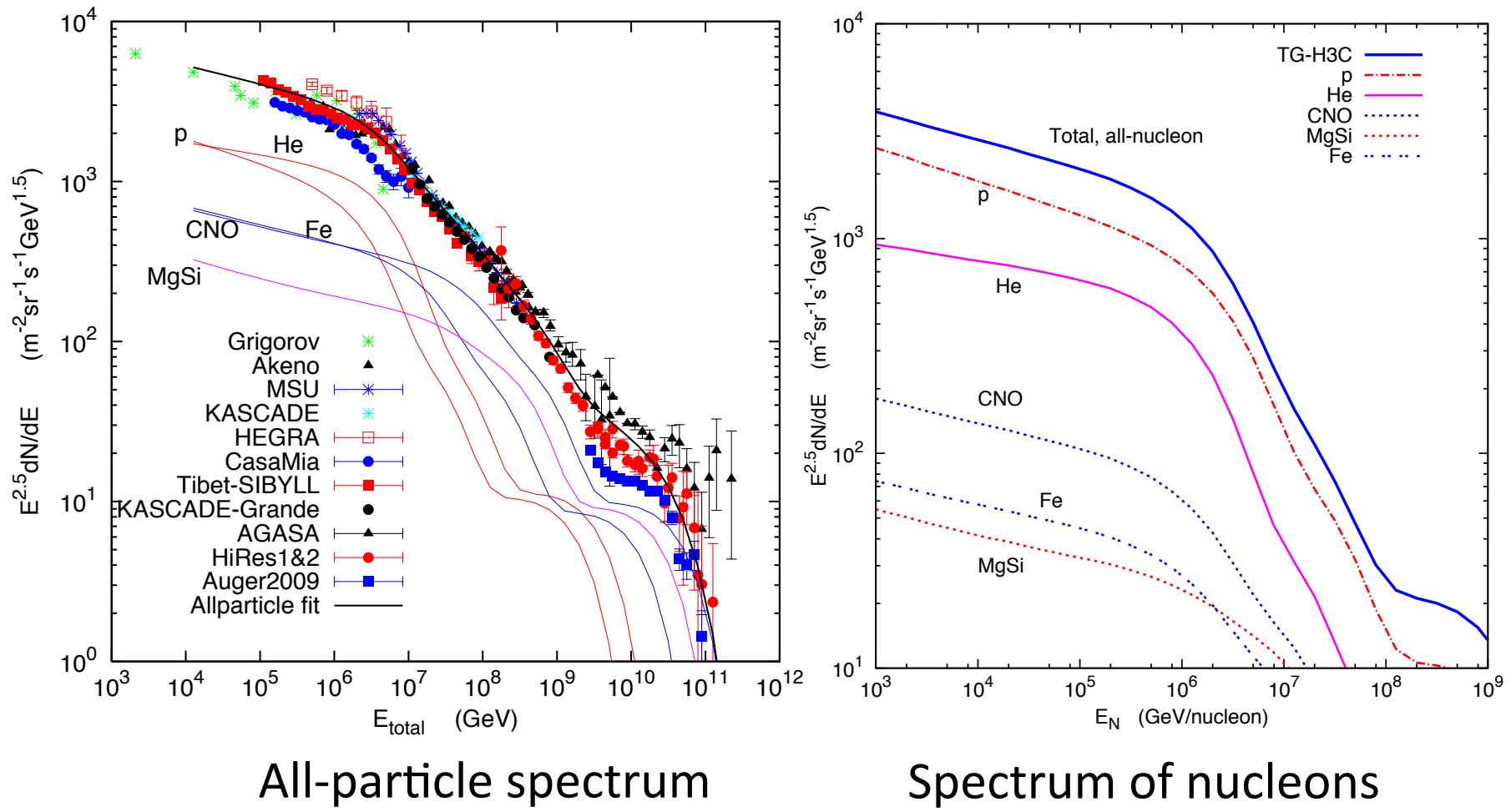
- $\text{Flux}(\nu) < \text{Flux}(\mu)$
- Kaons more important for ν than for μ
- Charm contribution same for ν and μ
 - Therefore charm relatively more important for ν
 - Charm contribution isotropic (compared to secant θ effect for $>\text{TeV}$ leptons from decay of π and K)



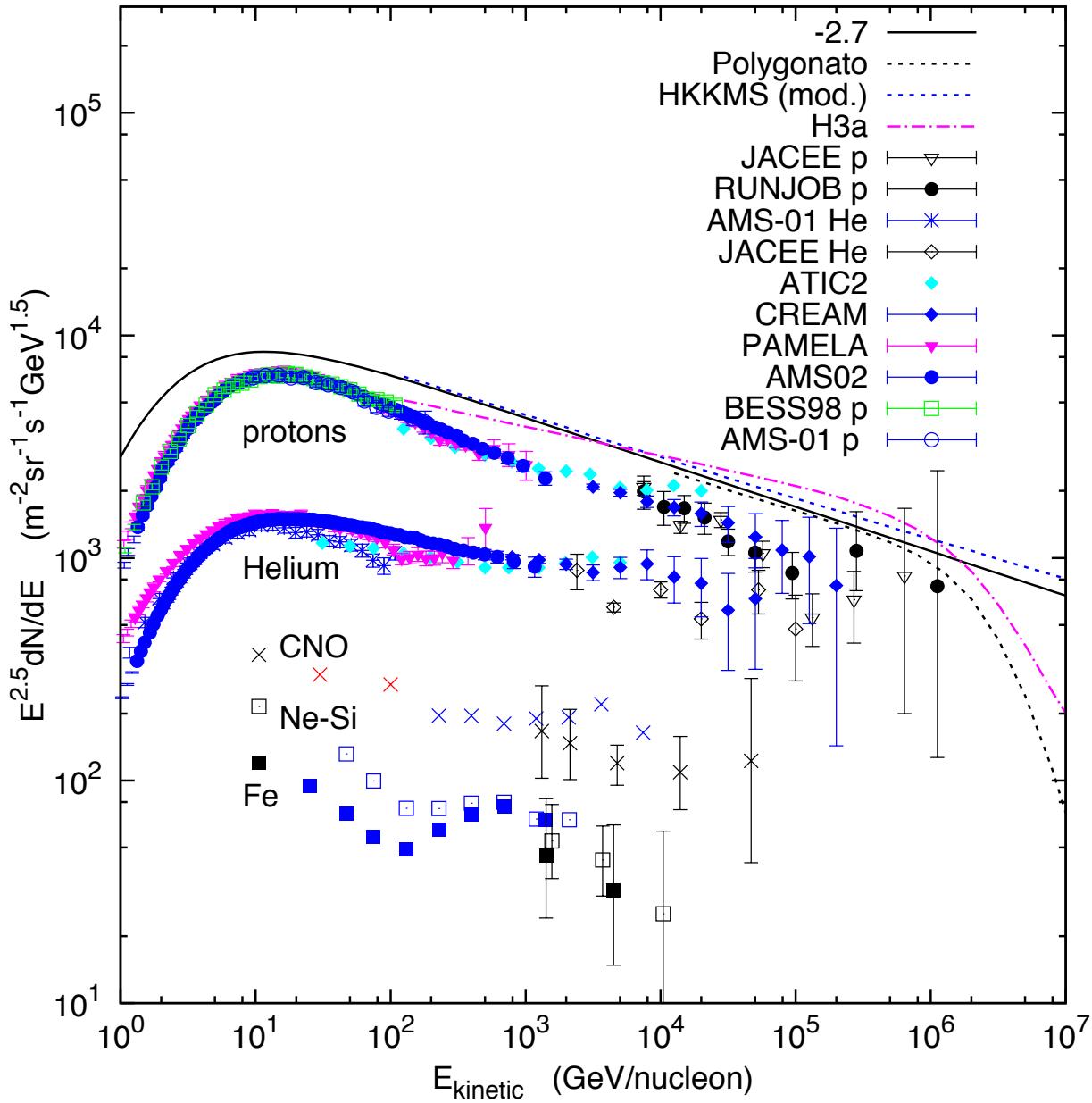
Primary spectrum

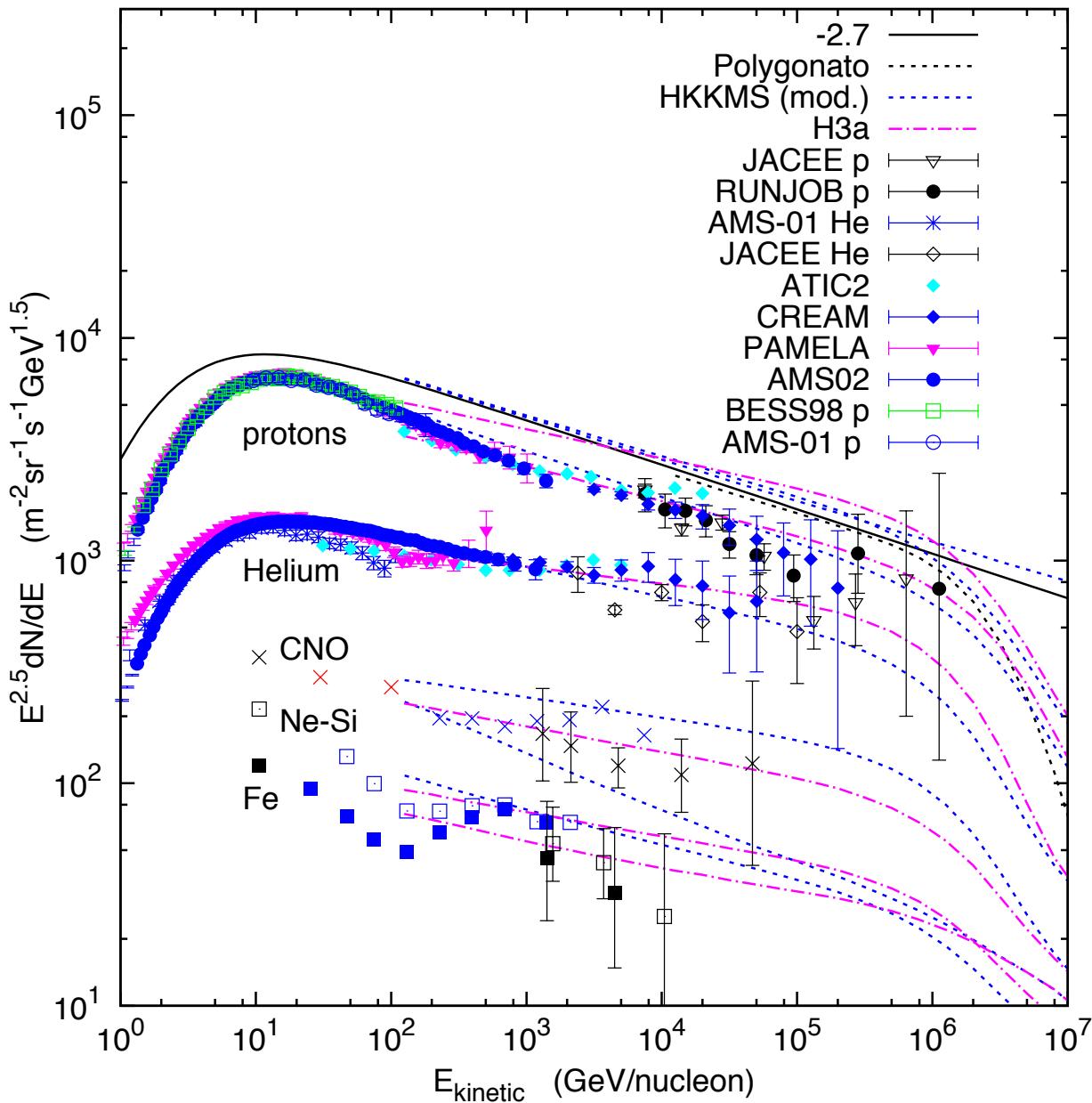
- Combine information
 - from direct measurements < 100 TeV
 - with air shower measurements of all-particle spectrum at higher E
- Assumptions:
 - 5 nuclear groups: p, He, CNO, Mg-Si, Fe
 - 3 populations: SNR, Hillas' Galactic component B, extra-galactic
 - All features depend on rigidity, $R = P_c / Z_e$
 - All particle spectrum: $\phi_i(E) = \sum_{j=1}^3 a_{i,j} E^{-\gamma_{i,j}} \times \exp\left[-\frac{E}{Z_i R_{c,j}}\right]$
 - Spectrum of nucleons: $\phi_{i,N}(E_N) = A \times \phi_i(A E_N)$
- Requirements
 - Consistency with air shower measurements of the all-particle spectrum
 - Anchor to composition from direct experiments below 100 TeV

Spectrum of nucleons determines fluxes of atmospheric neutrinos

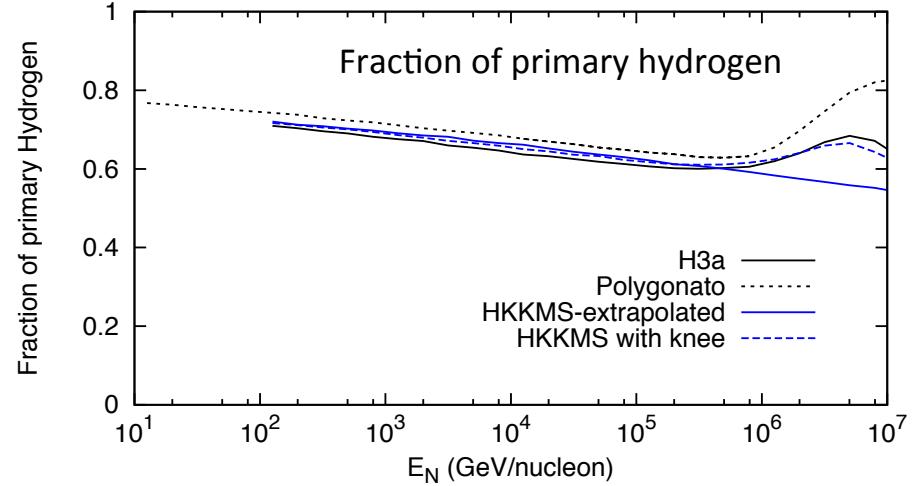
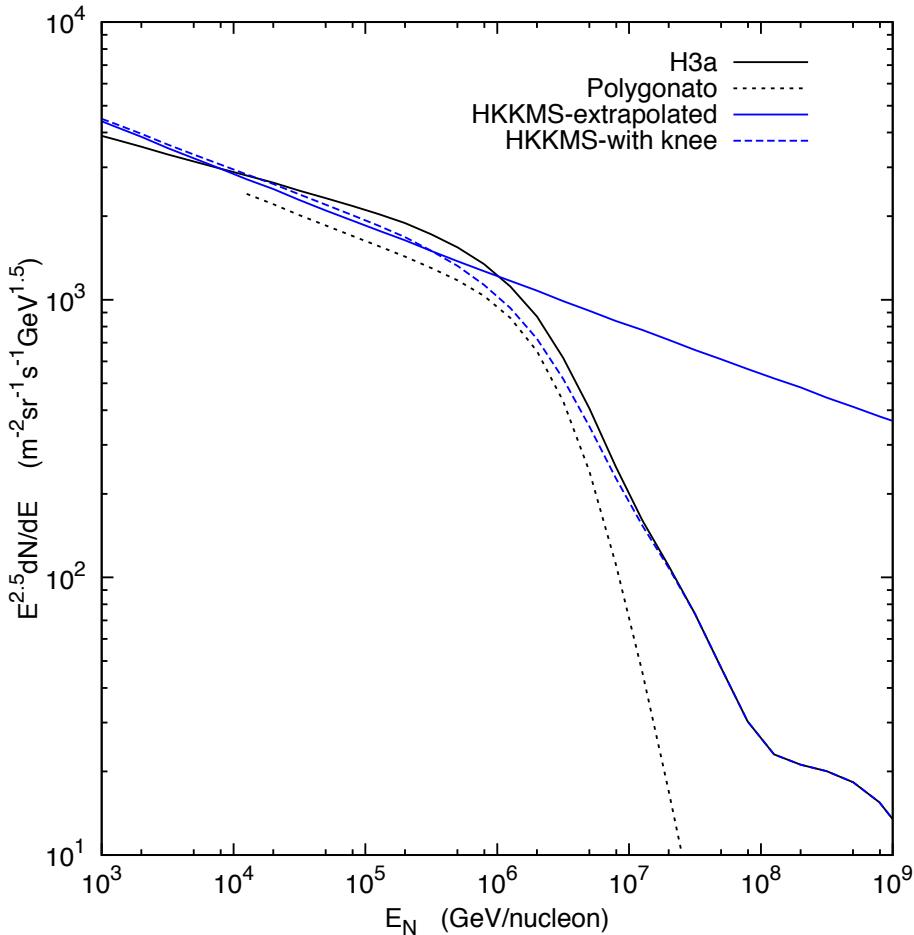


Direct Measurements





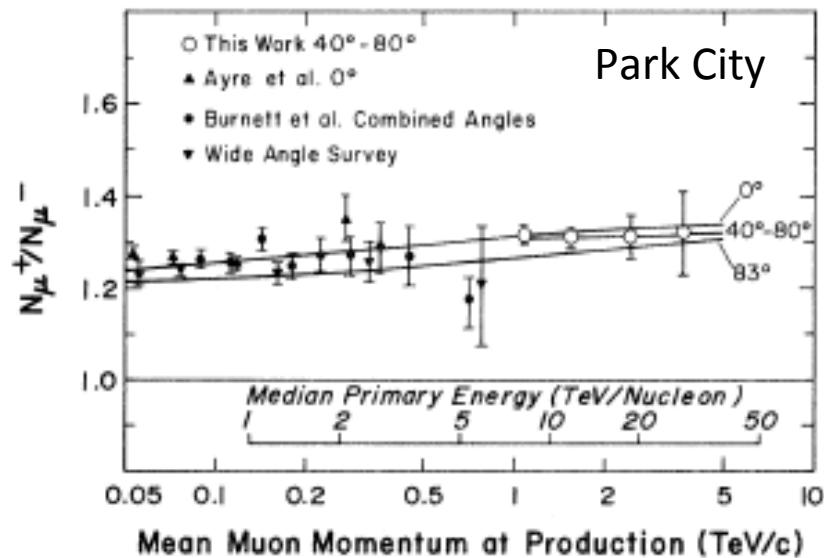
Spectrum of nucleons & δ_0



$$\delta = \frac{p - n}{p + n} \approx \frac{H}{\text{all}} \quad \text{needed to calculate } \mu^+ / \mu^-$$

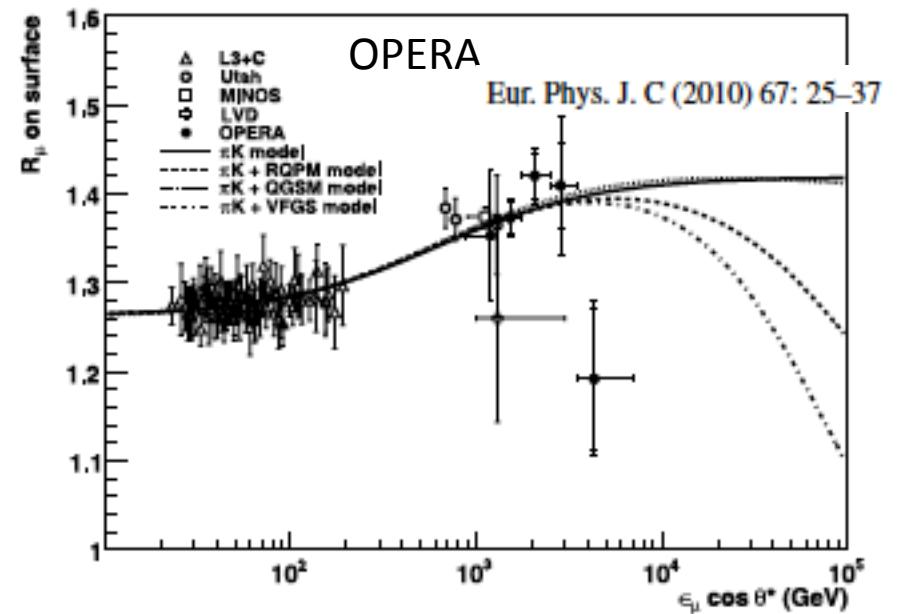
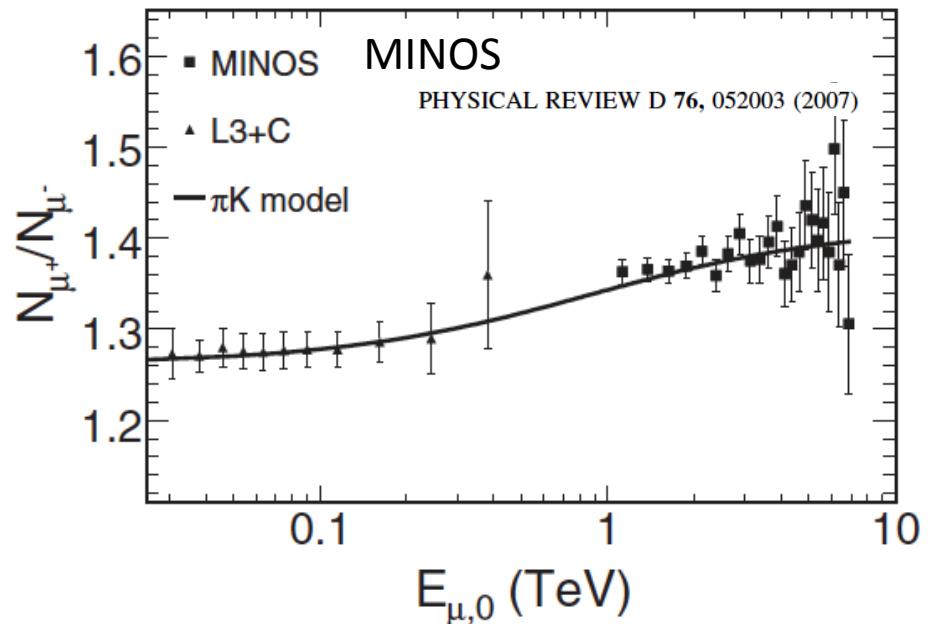
HKKMS = primary spectrum of nucleons used by Honda et al., Phys. Rev. D75, 043006 (2007)

Muon charge ratio



Ashley, Elbert, Keuffel, Larsen, Morrison, PRL 31(1973) 1091

- Ratio due to excess of p over n in primary CR + steep spectrum which favors $p \rightarrow \pi^+$ over $p \rightarrow \pi^-$
- Rise at TeV due to increased importance of Kaons (especially K⁺)



Follow the charges

Reference: TKG Astropart. Phys. 35 (2012) 801

$$\phi_N(E) = \phi_N(0) \times \exp\left(-\frac{X}{\Lambda_N}\right) \quad \Delta(X) = \delta_0 \phi_N(0) \times \exp\left(-\frac{X}{\Lambda_-}\right)$$
$$N = p + n \quad \delta_0 = \frac{p(0) - n(0)}{p(0) + n(0)} \quad \frac{1}{\Lambda_-} = \frac{1 - Z_{pp} + Z_{pn}}{1 + Z_{pp} + Z_{pn}} \frac{1}{\Lambda_N}$$

For the pion channel only, the result is (Frazer, 1972)

$$\phi_\mu(E_\mu)^\pm = \phi_N(E_\mu) \frac{A_{\pi\mu} \times 0.5(1 \pm \beta\delta_0\alpha_\pi)}{1 + B_{\pi\mu}^\pm E \cos(\theta)E_\mu/\epsilon_\pi}$$

$$\beta = \frac{1 - Z_{pp} - Z_{pn}}{1 - Z_{pp} + Z_{pn}} \approx 0.909 \quad \alpha_\pi = \frac{Z_{p\pi^+} - Z_{p\pi^-}}{Z_{p\pi^+} + Z_{p\pi^-}} \approx 0.165 \quad \alpha_K = \frac{Z_{pK^+} - Z_{pK^-}}{Z_{pK^+} + Z_{pK^-}}$$

Kaon channel

$$\phi_K(E_\mu)^- = \frac{Z_{NK^-}}{Z_{NK}} \phi_N(E_\mu) \frac{A_{NK}}{1 + B_{K\mu} \cos(\theta) E_\mu / \epsilon_K}.$$

$$\phi_K(E_\mu)^+ = \phi_N(E_\mu) A_{NK} \times \frac{\frac{1}{2}(1 + \alpha_K \beta \delta_0)}{1 + B_{K\mu}^+ \cos(\theta) E_\mu / \epsilon_K}$$

$$p \rightarrow n \pi^+ = n \rightarrow p \pi^- \text{ but } p \rightarrow \Lambda K^+ = n \rightarrow \Lambda K^0 \qquad \alpha_K = \frac{Z_{pK^+} - Z_{pK^-}}{Z_{pK^+} + Z_{pK^-}}$$

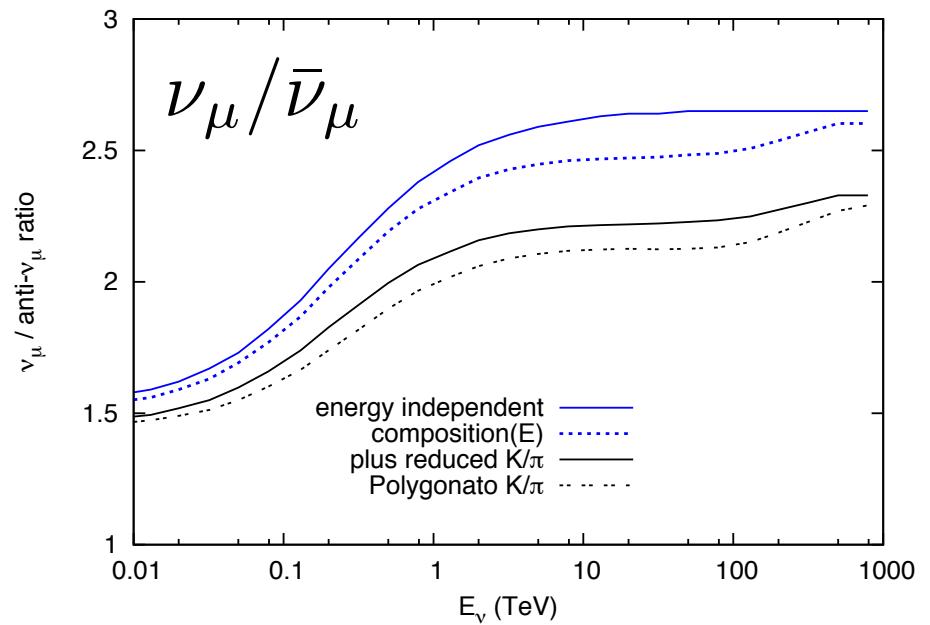
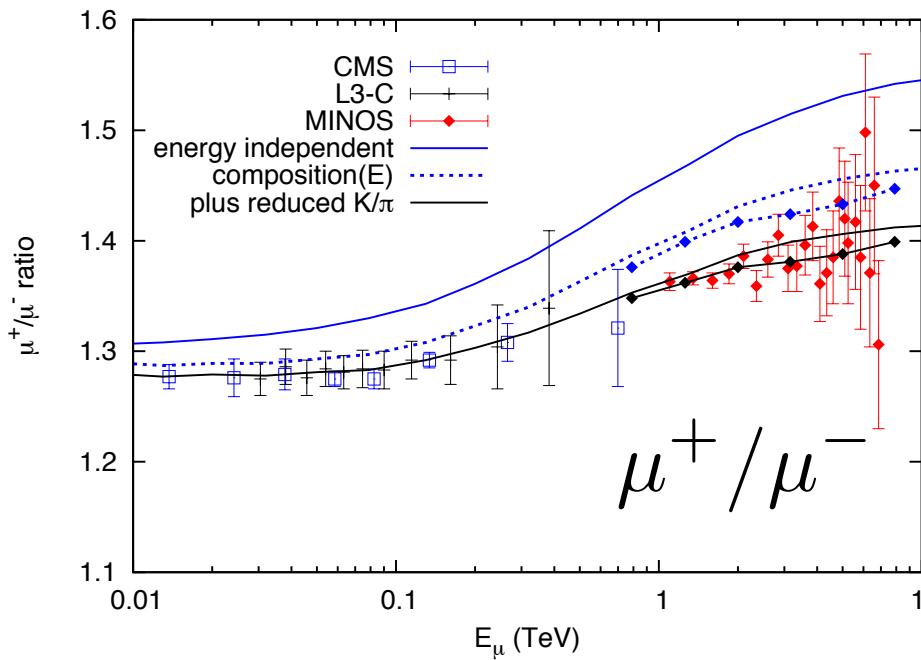
To evaluate the formulas we need the proton excess, $\delta_0 = \frac{p(0) - n(0)}{p(0) + n(0)}$

and we need the energy spectrum of nucleons $\phi_N(E_N)$ per GeV/nucleon

Results for Z_{pK^+}

$$Z_{p \rightarrow K^+} = 0.0090 \\ \rightarrow 0.0079$$

$$R_{K/\pi} \equiv \frac{Z_{p \rightarrow K^+} + Z_{p \rightarrow K^-}}{Z_{p \rightarrow \pi^+} + Z_{p \rightarrow \pi^-}} = 0.149 \\ \rightarrow 0.135$$



$p \rightarrow \Lambda K^+$ is relatively more important for $\nu/\bar{\nu}$

How to account for the knee in $\phi_\nu(E_\nu)$

1. Calculate $\int_{E_\nu}^{\infty} \phi_n(E_0) Y(E_\nu, E_0) dE_0$
 - a. Use full Monte Carlo (problem with statistics)
 - b. Or use an approximation for the yield of neutrinos
2. Integrate the cascade equations
 - a. Accounting for energy dependence at each step
 - b. Or modify the scaling equations assuming changes in spectral index and scaling violations are smooth

$$A_{N\pi^\pm} \sim \int_0^1 \frac{dx}{x^2} \frac{\phi_N(E/x)}{\phi_N(E)} \frac{dn_{N\pi^\pm\nu}(E/x, E)}{dE}$$

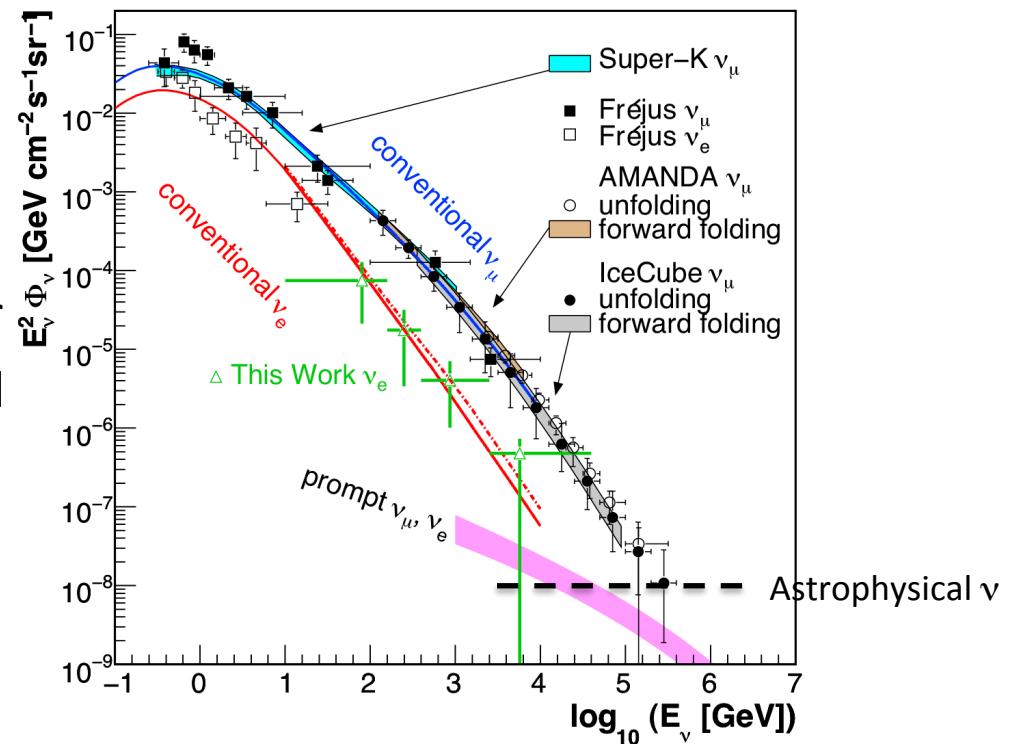
Refs.: Thunman, Ingelman, Gondolo, Astropart. Phys. 5 (1996) 309.

Fedynitch, Becker Tjus, Desiati, PRD 86(2012) 114014

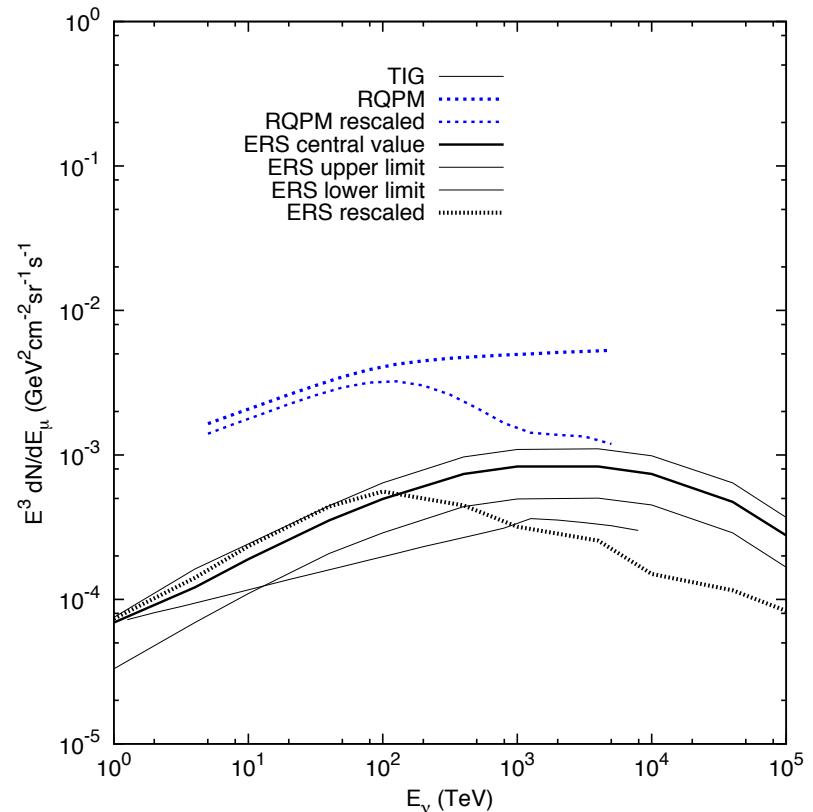
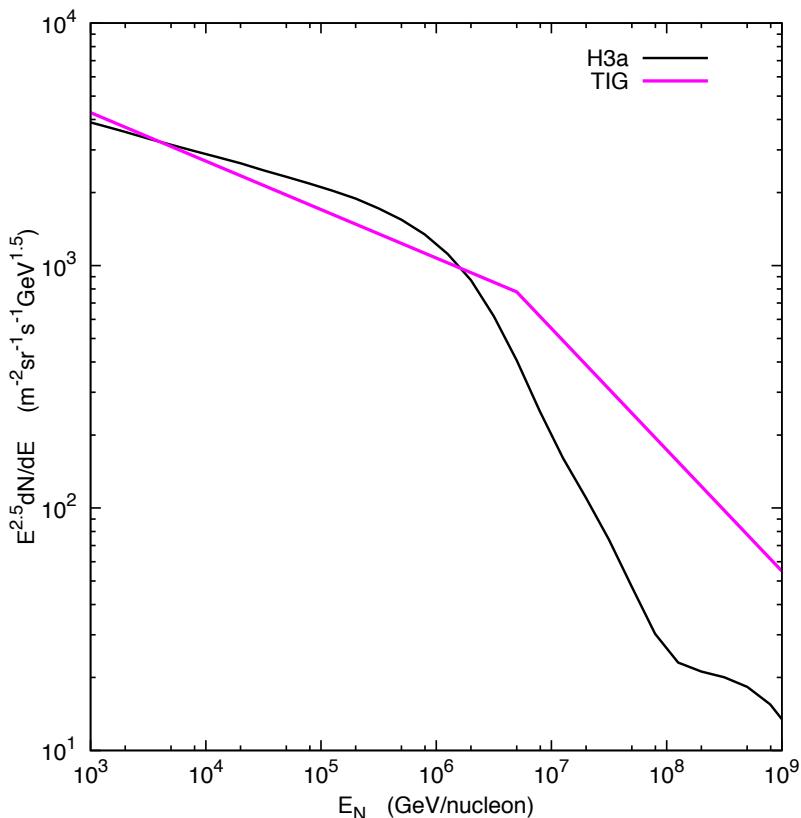
TG, arXiv:1303.1431

Importance of charm

- Critical energy $\epsilon_{\text{charm}} \approx 10^7$ GeV
- So spectrum of ν from charm follows primary spectrum
- Conventional ν one power steeper
- Crossover of prompt/conventional competes with the transition to astrophysical neutrinos



Charm models

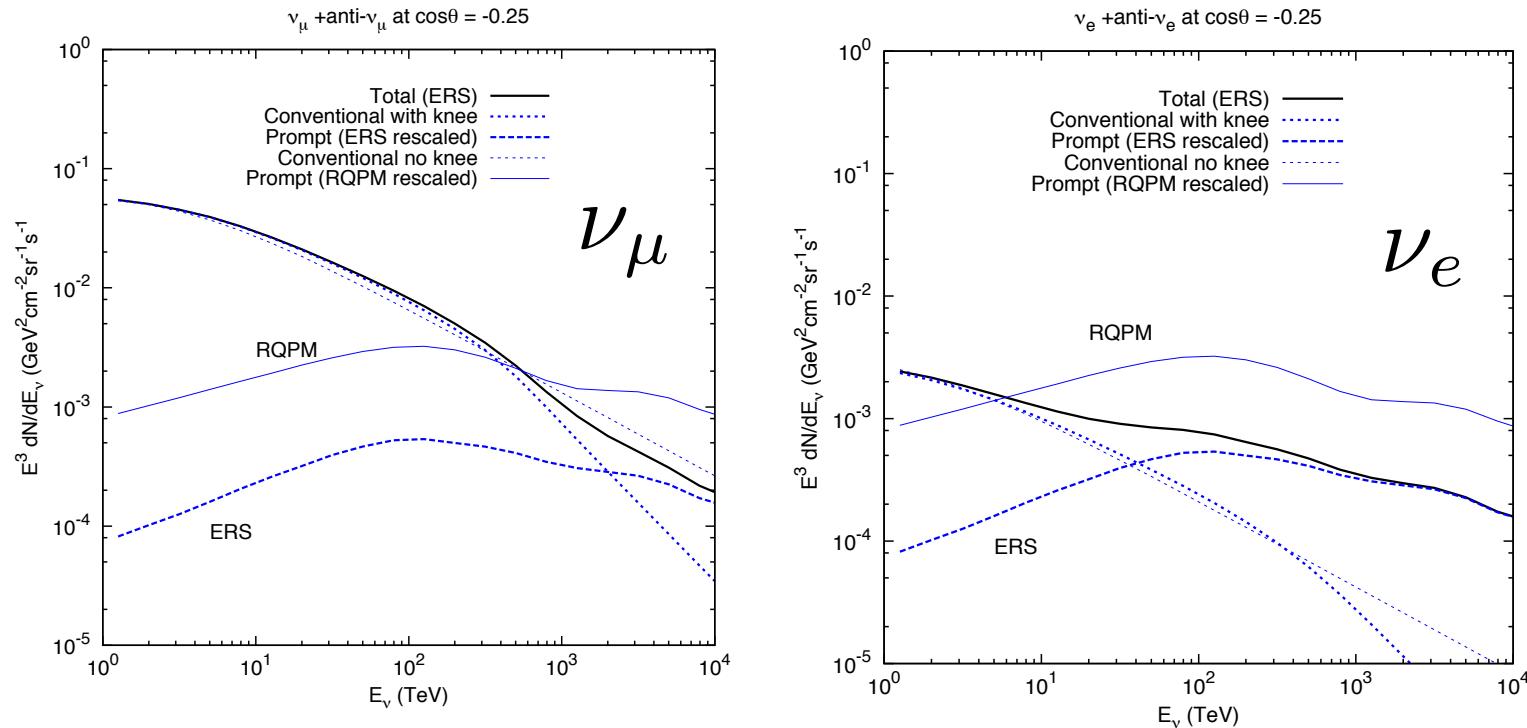


Compare two charm models:

QCD model: ERS = Enberg, Reno, Sarcevic, PRD 78 (2008) 043005 (uses TIG primary spectrum)

RQPM includes intrinsic charm: Bugaev et al., N.C. C12 (1989) 41, PRD 58 (1998) 054001

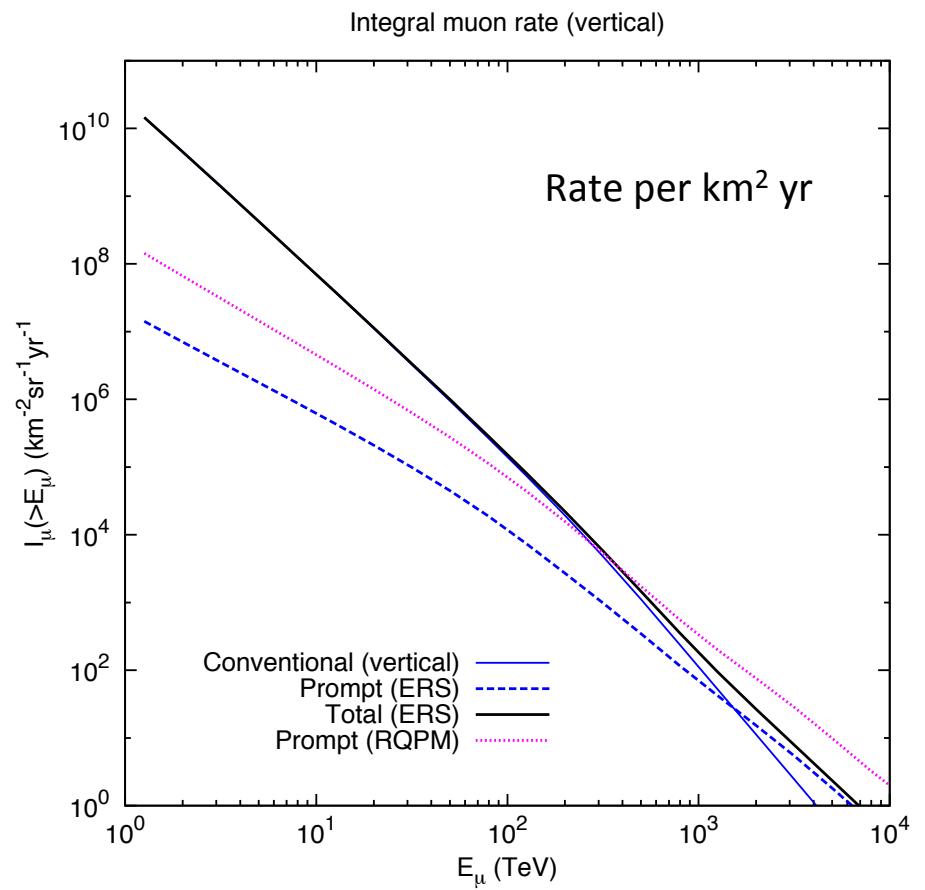
Compare ν_μ and ν_e fluxes



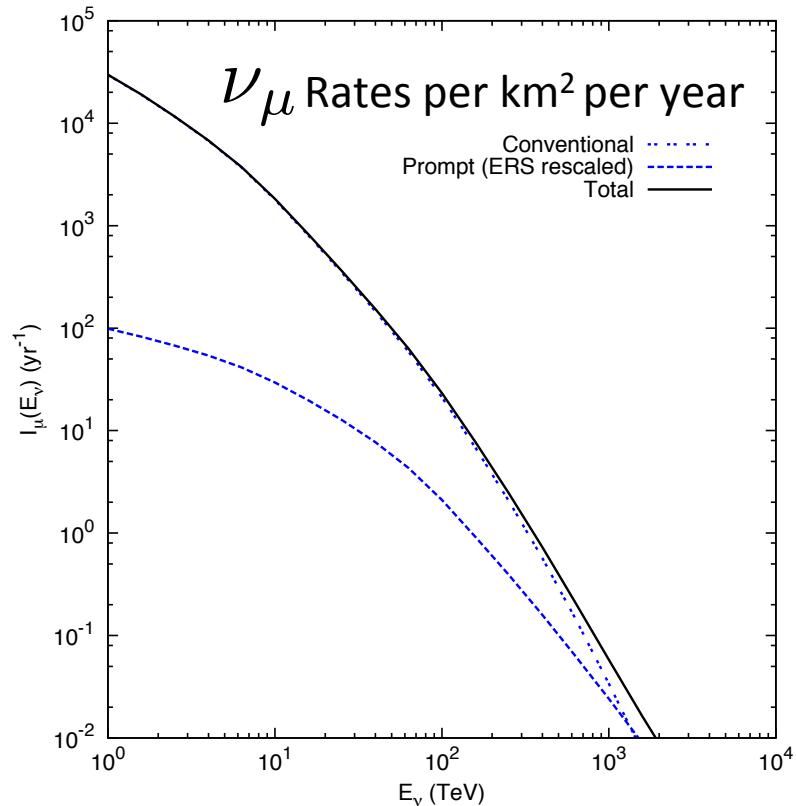
- Prompt crossover occurs at lower energy for ν_e
- Prompt component should be easier to see in cascade events

Can we identify prompt component with atmospheric muons?

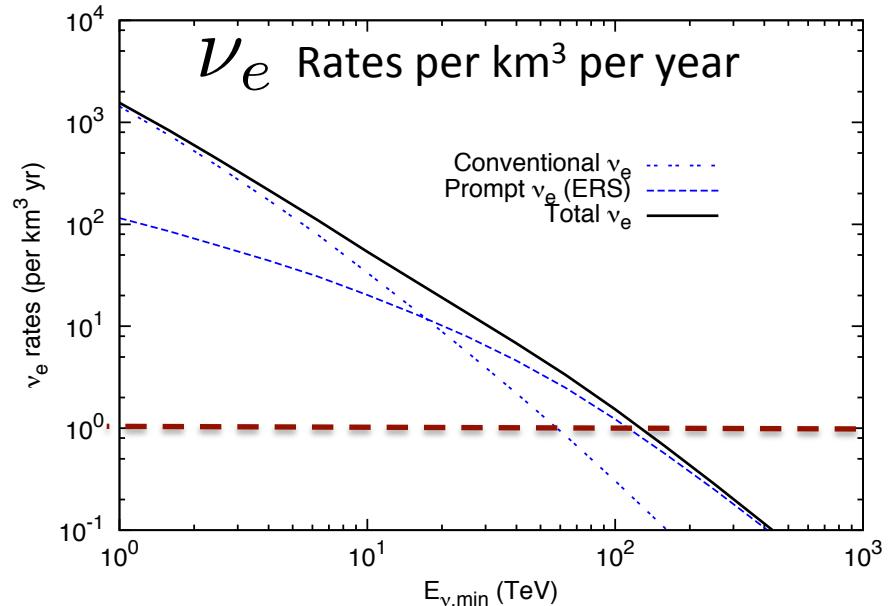
- Advantages:
 - No astro component
 - High rate
 - Angular dependence
 - Isotropic for prompt
 - $\sec(\theta)$ for conventional
 - Seasonal variation
 - Strong for conventional
 - Absent for prompt
- Problems in practice
 - Crossover at high energy
 - Energy resolution



Compare ν_μ -induced μ with ν_e



ν_μ : high rates but crossover at high energy and E_μ is measured (not E_ν)
(Chris Weaver's talk)

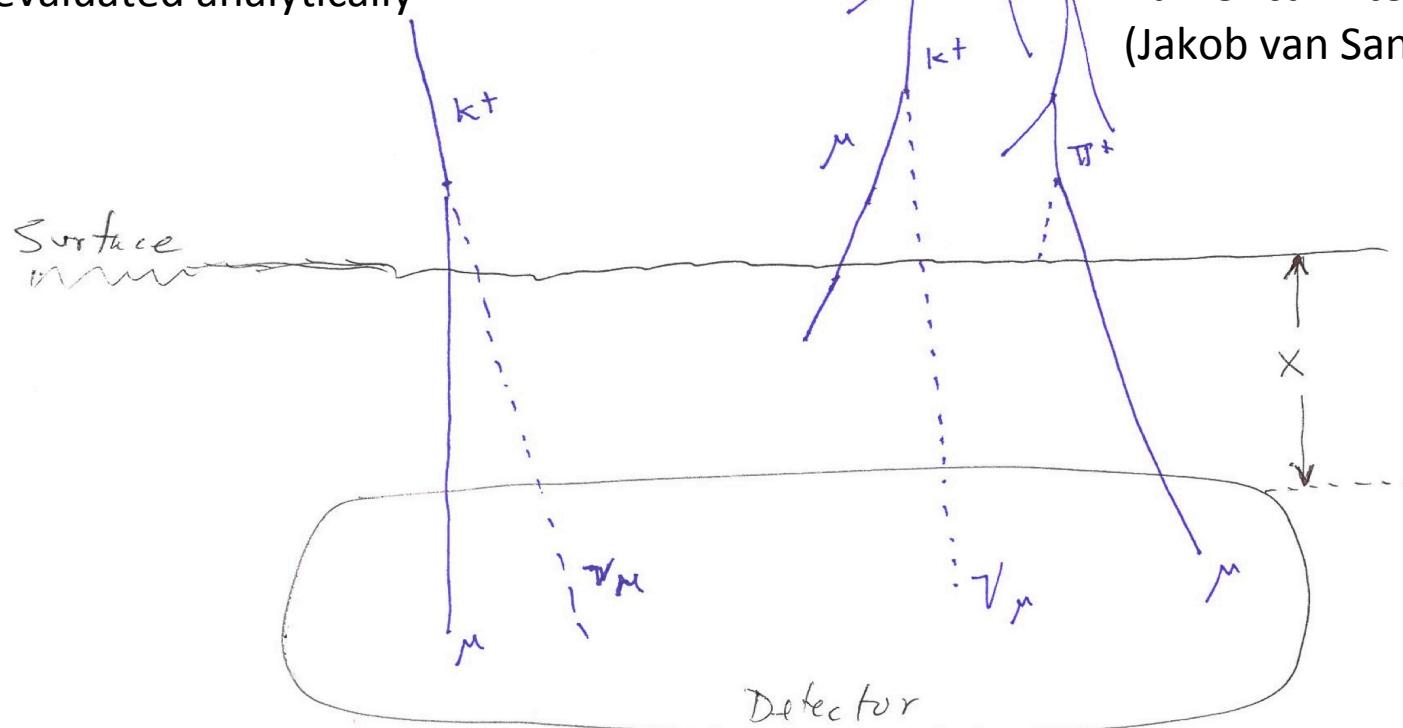


- ν_e : Rates lower, but
- Crossover at lower energy
 - Self veto for a fraction of atmospheric events
 - Astro component easier to see
(Jakob van Santen's talk)

Atmospheric neutrino self veto

Two cases

1. Stefan Schönert et al.
Phys. Rev. D79 (2009) 043009
Can be evaluated analytically



2. Veto by an unrelated μ
--also applies to ν_e
Requires Monte Carlo or numerical integration
(Jakob van Santen's talk)

Angular/energy dependence

- Analytic calculation:
 - For $E_\nu > 100$ TeV
 - Passing rate < 10% for $\cos\theta > 0.3$
 - 70% of downward phase space
 - Even better at higher E
- Generic veto seems to be comparable
 - Applies to ν_e

